

# Calculus-I

## Specific Objectives of the Course:

Calculus is serving as the foundation of advanced subjects in all areas of mathematics. The course, equally, emphasizes the basic concepts and skills needed for mathematical manipulation. This Calculus focus on the study of functions of a single variable.

## Course Outline:

Functions, upper and lower bounds of variables and functions, inverses of exponential, circular, hyperbolic and logarithmic functions, one sided and two sided limits of functions, continuity of functions and their graphical representations, properties of continuous function on closed bounded intervals, discontinuity of function and its types. Derivatives: Definition, techniques of differentiation. Derivatives of polynomials and rational, exponential, logarithmic and trigonometric functions, Inverse functions and their derivatives. The chain rule. Implicit differentiation. Rates of change in natural and social sciences. Related rates. Linear approximations and differentials. Higher derivatives, Leibnitz's theorem. Applications of derivatives: Increasing and decreasing functions. Relative extrema and optimization. First derivative test for relative extrema. Convexity and point of inflection. The second derivative test for extrema. Indeterminate forms and L'Hopitals rule. Anti-derivatives and integrals. Riemann sums and the definite integral. Properties of Integral.

## Recommended Books:

- J. Stewart, Calculus (5th edition or latest edition), 2002, Brooks/Cole
- H. Anton, I. Bevens, S. Davis, Calculus: A New Horizen (8th edition or latest), 2005, John Wiley, New York
- G. B. Thomas, A. R. Finney, Calculus (11th edition or latest edition), 2005, Addison-Wesley, Reading, Ma, USA

## Elements of Set Theory and Mathematical Logic

**Specific Objectives of course:** Everything mathematicians do can be reduced to statements about sets, equality and membership which are basics of set theory. This course introduces these basic concepts. The course aims at familiarizing the students with cardinals, relations and fundamentals of propositional and predicate logics.

**Course Outline:** Set theory: Sets, subsets, operations with sets: union, intersection, difference, symmetric difference, Cartesian product and disjoint union.

Functions: graph of a function. Composition; injections, surjections, bijections, inverse function.

Computing cardinals: Cardinality of Cartesian product, union.

Mathematical Logic: Introduction, statements, negation, logical connectives, truth table, conjunction, disjunction, conditional and Bi-conditional statements, and converse, inverse and contrapositive statements, Tautology and contradiction, Predicates and quantifiers, Cardinality of all injective, surjective and bijective functions from a set to another set. Infinite sets, finite sets. Countable sets, introduction to first and second countable, properties, examples  $(\mathbb{Z}, \mathbb{Q})$ .  $\mathbb{R}$  is not countable.  $(\mathbb{R}, \mathbb{R} \times \mathbb{R}, \mathbb{R} \times \mathbb{R} \times \mathbb{R})$ , have the same cardinal. Operations with cardinal numbers. Cantor-Bernstein theorem.

Relations: Equivalence relations, partitions, quotient set; examples, parallelism, similarity of triangles. Order relations, min, max, inf, sup; linear order. Examples:  $\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathcal{P}(A)$ . Well -ordered sets and induction. Inductively ordered sets and Zorn's lemma. Propositional Calculus. Truth tables. Predicate Calculus.

### Recommended Books:

- M. Liebeck, A Concise Introduction to Pure Mathematics, CRC Press, 2011.
- N. L. Biggs, Discrete Mathematics, Oxford University Press, 2002.
- R. Garnier, J. Taylor, Discrete Mathematics, Chapters 1,3,4,5, CRC Press, 2010.
- A.A. Fraenkel, Abstract Set Theory, North-Holland Publishing Company, 1966.
- P. Suppes, Axiomatic Set Theory, Dover Publication, 1972.
- P.R. Halmos, Naive Set Theory, New York, Van Nostrand, 1950.
- B. Rotman, G.T. Kneebone, The Theory of sets and Transfinite Numbers, Oldbourne London, 1968.
- D. Smith, M. Eggen, R.St. Andre, A Transition to Advanced Mathematics, Brooks/Cole, 2001.

## **Discrete Mathematics**

### **Specific Objectives of the Course:**

This course shall assume background in number theory. It lays a strong emphasis on understanding and utilizing various strategies for composing mathematical proof.

### **Course outline:**

Basic methods: product, inclusion-exclusion formulae. Permutations and combinations. Recurrence relations and their solutions. Generating functions. Double counting and applications. Pigeonhole principle and applications. Binary relations, n-ary Relations. Closures of relations. Composition of relations, inverse relation. Graph terminology. Representation of graphs. Graphs isomorphism.

Algebraic methods: the incidence matrix. Connectivity, Eulerian and Hamiltonian paths. Shortest path problem. Trees and spanning trees. Complete graphs and bivalent graphs.

Boolean algebra: Introduction to gates and its types, combinatorial circuit of graphs, Boolean expression, Boolean function and its representation.

Graph theory: order and types. Matrix representation of a graph, Graph isomorphism.

### **Recommended Books:**

- S. E. Susana, Discrete Mathematics with applications
- D. P. Acharjya, Sreckummar, Discrete Mathematics
- K. H. Rossen, Discrete Mathematics and its applications
- J. Gersting, Mathematical Structures for Computer Sciences

## Calculus II

**Specific Objectives of course:** This is second course of Calculus. As continuation of Calculus I, it focuses on techniques of integration and applications of integrals. The course also aims at introducing the students to infinite series, parametric curves and polar coordinates.

### Course Outline:

**Techniques of integration:** Integrals of elementary, hyperbolic, trigonometric, logarithmic and exponential functions. Integration by parts, substitution and partial fractions. Approximate integration. Improper integrals. Gamma functions.

**Applications of integrals:** Area between curves, average value. Volumes. Arc length. Area of a surface of revolution. Applications to Economics, Physics, Engineering and Biology.

**Infinite series:** Sequences and series. Convergence and absolute convergence. Tests for convergence: divergence test, integral test, pseries test, comparison test, limit comparison test, alternating series test, ratio test, root test. Power series. Convergence of power series. Representation of functions as power series. Differentiation and integration of power series. Taylor and McLaurin series. Approximations by Taylor polynomials.

### Conic section, parameterized curves and polar coordinates:

Curves defined by parametric equations. Calculus with parametric curves: tangents, areas, arc length. Polar coordinates. Polar curves, tangents to polar curves. Areas and arc length in polar coordinates.

### Recommended Books:

1. Thomas, *Calculus*, 11th Edition. Addison Wesley Publishing Company, 2005
2. H. Anton, I. Bevens, S. Davis, *Calculus*, 8th Edition, John Wiley & Sons, Inc. 2005
3. Hughes-Hallett, Gleason, McCallum, et al, *Calculus Single and Multivariable*, 3rd Edition. John Wiley & Sons, Inc. 2002.
4. Frank A. Jr, Elliott Mendelson, *Calculus*, Schaum's outlines series, 4th Edition, 1999
5. C.H. Edward and E.D Penney, *Calculus and Analytic Geometry*, Prentice Hall, Inc. 1988
6. E. W. Swokowski, *Calculus with Analytic Geometry*, PWS Publishers, Boston, Massachusetts, 1983.
7. M. Liebeck, *A Concise introduction to pure Mathematics*, CRC Press, 2011.
8. A. Kaseberg, *Intermediate Algebra*, Thomson Brooks/COLE, 2004.
9. J. Stewart, *Calculus early transcendentals*, 7th Edition, Brooks/COLE, 2008.

## **Algebra-I (Group Theory)**

### **Specific Objectives of the Course:**

This is the first course in groups, matrices and linear algebra, which provides basic background needed for all mathematics majors, is a prerequisite for many courses. Many concepts presented in the course are based on the familiar setting of plane and real three-space, and are developed with an awareness of how linear algebra is applied.

### **Course Outline:**

Basic axioms of a group with examples, subgroups, order of a group, subgroups generated by subset of a group, system of generators cyclic groups, cosets, Lagrange's theorem, introduction to permutations, even and odd permutations, cycles, lengths of cycles, transpositions, symmetric group, alternating groups. Preliminaries, normalizers and centralizers of a group, center of a group, normal subgroup, quotient groups, conjugacy relation between elements and subgroups, homomorphism and isomorphism between groups, homomorphism and isomorphism theorems, finite p-groups, internal and external direct products, endomorphism and automorphism of a group, characteristic and fully invariant subgroups, direct product of groups.

### **Recommended Books:**

- E. Arnold, Rings, Fields and Groups: An Introduction to Abstract Algebra, 1983
- A. Majeed, Group Theory, Ilmi kitab Khana
- J. B. Farleigh, A First Course in Abstract Algebra (7th edition), Addison-Wesley
- I. D. Macdonald, The Theory of Groups, 1975, Oxford Clarendon Press, Ma., USA
- K. H. Dar, Abstract Algebra, ilmi Kitab Khana Lahore.

## Calculus-III

### Specific Objectives of the Course:

Multivariate calculus is serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes the basic concepts and skills needed for mathematical manipulation. The main focus will be on the study of functions having two, three or more variables. Course Outline: Function of several variables, limit and continuity, derivability and differentiability, chains rules, properties of several variable function, Euler's theorem, total differentials and explicit and implicit functions, extreme values: maxima and minima with or without constraints, Taylor's theorem in multi variable and its various form, chain of variables, functions of functions, double and triple integrals with applications (area of surfaces and volumes of revolution using double and triple integrals) line integrals, integration on  $\mathbb{R}^2$  : integrals over a region, the Green's, the divergence and the Stokes theorems with applications.

### Recommended Books:

- J. Stewart, Calculus (5th edition or latest edition), 2002, Brooks/Cole (suggested text)
- H. Anton, I. Bevens, S. Davis, Calculus: A New Horizen (8th edition or latest), 2005, John Wiley, New York
- G. B. Thomas, A. R. Finney, Calculus (11th edition or latest), 2005, Addison-Wesley, Reading, Ma, USA

## **Computer Programming**

**Credit Hours: 2 + 1**

### **Specific Objectives of the Course:**

The purpose of this course is to introduce students to operating systems and environments.

### **Course Outline:**

Introduction to programming, applications of programming in mathematics, program structure, flow chart, C/C++ language, building blocks, variables, data types, input/output, repetition (FOR, WHILE, DO), selection (IF, IF ELSE, ELSE IF) construct switch statement, conditional statement, function that returns a value using argument to pass data to another function, external variable, arrays and strings, pointers, structure, file processing and introduction to object-oriented programming.

### **Recommended Books:**

- Dietel & Dietel, C++ How to program, 7th Edition, Prentice Hall
- H. Schildt, C/C++ The Complete Reference, 4th Edition, McGraw Hill Osborne media
- J. L. Hein, Theory of Computations: An Introduction, Jones and Bartlett, Boston
- R. Laffore, Introduction to Object Programming, McGraw Hill, New York

## Affine Euclidean Geometry

### Specific Objectives of course:

To familiarize mathematics students with the axiomatic approach to geometry from a logical, historical, and pedagogical point of view and introduce them with the basic concepts of Affine Geometry, Affine spaces and Platonic Polyhedra.

### Course Outline:

Vector spaces and affine geometry: Collinearity of three points, ratio  $AB/BC$ . Linear combinations and linear dependent set versus affine combinations and affine dependent sets.

Classical theorems in affine geometry: Thales, Menelaus, Ceva, Desargues. Affine subspaces, affine maps. Dimension of a linear subspace and of an affine subspace.

Euclidean geometry: Scalar product, Cauchy-Schwartz inequality: norm (magnitude) of a vector, distance between two points, angles between two non-zero vectors. Pythagoras theorem, parallelogram law, cosine and sine rules. Elementary geometric loci.

Orthogonal transformations: Isometries of plane (four types), Isometries of space (six types). Orthogonal bases.

Platonic polyhedra: Euler theorem on finite planar graphs. Classification of regular polyhedral in space. Isometries of regular polygons and regular polyhedra.

### Recommended Books:

- E. Rees, Notes on Geometry, Springer, 2004.
- M. A. Armstrong, Groups and Symmetry, Springer, 1998.
- H. Eves, Fundamentals of Modern Elementary Geometry, Jones and Bartlett Publishers International, 1992
- S. Stahl, The Poincare Half-Plane A Gateway to Modern, Geometry, Jones and Bartlett Publishers International, 1993



## Linear Algebra

### Specific Objectives of the Course:

This is a course in abstract linear algebra. The majority of follow up courses in both pure and applied mathematics assume the material covered in this course.

**Course Outline:** Algebra of matrices, determinants, matrix of a linear transformation, row and column operations, rank, inverse of matrices, solution of homogeneous and non-homogeneous equations, orthogonal transformation. Vector spaces, subspaces, linear dependence and independence, linear span of a subset of a vector space, bases and dimensions of a vector space, sums and direct sums of subspaces of a finite dimensional vector space, dimension theorem, linear transformation, null space, image space of linear transformation, rank and nullity of a linear transformation, relation between rank, nullity and dimension of the domain of a linear transformation, matrix of linear transformation, change of basis, inner product spaces, orthogonal and orthonormal basis, similar matrices and diagonalization of a matrix,  $\text{Home}(V,W)$ , dimension and basis of  $\text{Home}(V,W)$ , dual space and dual basis, annihilators, Eigen values and Eigen vectors and minimal polynomials.

### Recommended Books:

- S. J. Axle, Linear Algebra Done Right, Undergraduate Texts in Mathematics, 1996, Springer, New York, Schaum's outlines series
- G. Birkhoff, S. Maclane, A Survey of Modern Algebra (4th edition), AKP
- W. L. C. Perry, Elementary Linear Algebra, 1988, McGraw-Hill, New York